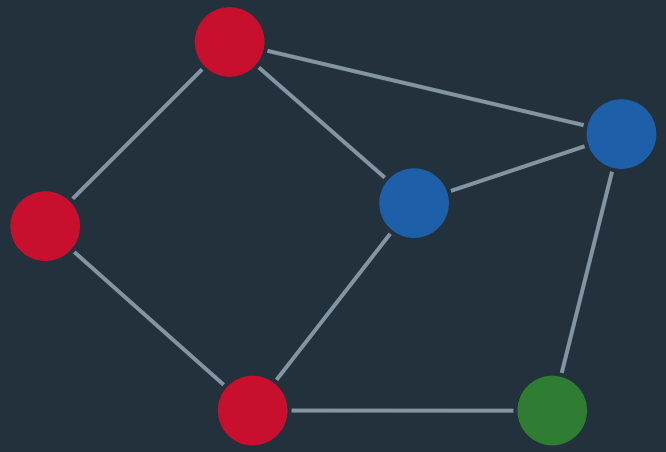
Describing a whole network



Course A · Week 4 · Chapter 6

One lecture and one lab, about 90 minutes each

Instructors are free to use, adapt and share these slides.

Week 4: four lecture blocks, a break, one lab

0:00	**1 Size and density**	20 min
0:20	**2 Pairs: the dyad census and reciprocity**	20 min
0:40	**3 Triples: transitivity and the triad census**	25 min
1:05	**4 Mixing: do ties stay within groups?**	25 min
1:30	Break	15 min
1:45	**5 Lab: run it, then check your answers**	90 min

Three data sets: one firm in the middle

```
. nwwebuse lazega, nwclear
. nwsummarize lazega_adv
```

Run first. Every example starts with nwwebuse …, nwclear.

Directed, 71 lawyers, 892 advice arcs. nwclear removes networks of earlier work.

Lazega: 71 lawyers

Advice, co-work and friendship; office (48, 19, 4 lawyers), status (35 associates, 36 partners), gender (53 men, 18 women), age, years with the firm.

London gang: 54

Co-offending ties; birthplace groups of 12, 6, 24 and 12 members.

Glasgow girls: 50

Friendship nominations in a school year group, wave 1 (glasgow1): 113 arcs.

Three expectations about the firm, three counts

Is advice returned?

Dyad census and reciprocity.

```
nwdyads lazega_adv
```

Does advice close into triangles?

Transitivity and the triad census.

```
nwtriads lazega_adv
```

Does advice stay within offices?

Mixing and the E-I index, read against group sizes.

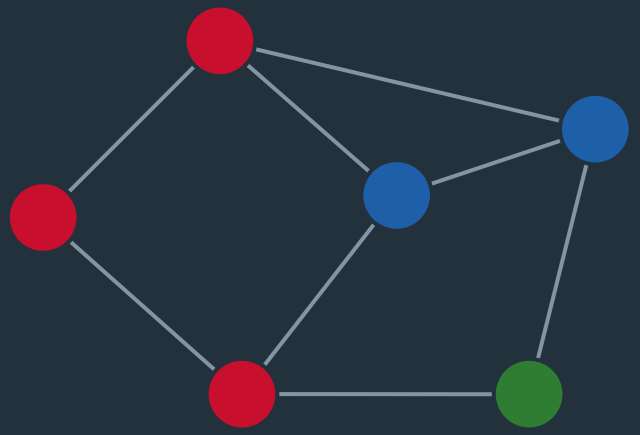
```
nwmixing lazega_adv
```

None of the counts says why. They describe how much exchange, closure and sorting there is.

1

Size and density

20 min including the opening · Chapter 6: density, mean degree, why networks of different size cannot be compared



Density: 892 arcs among 71 lawyers

```
. nwsummarize lazega_adv
   Nodes: 71
   Arcs: 892
   Density:  .179
   Mean outdegree:  12.56
```

.179

density of advice: 892 of 4,970 possible arcs

12.56

mean outdegree: advisors named per lawyer

Relation (71 lawyers)	Density	Mean outdegree
Advice, lazega_adv	.179	12.56
Co-work, lazega_cow	.222	15.55
Friendship, lazega_fr	.116	8.10

Same size, so comparable: friendship is the sparsest of the three relations, work the densest.

Each lawyer names 12.56 colleagues on average, out of 70 possible. The densities are the same information divided by 70.

A gap in density is not a gap in cohesion

```
. nwwebuse glasgow, nwclear
. nwsummarize glasgow1
   Nodes: 50
   Arcs: 113
   Density:   .0461
   Mean outdegree:   2.26
```

Network	Nodes	Arcs	Density	Mean outdegree
Lawyers, advice	71	892	.179	12.56
Glasgow girls, friendship, wave 1	50	113	.046	2.26

Three differences at once

Size (71 and 50), the meaning of the relation (advice and friendship), and the process that makes ties. The density gap mixes all three.

No ranking of cohesion

Density says how much of the possible is present. It does not say which group holds together better.

Report mean degree next to density

Nodes	Possible pairs	Typical ties per node at density .1
20	190	1.9
50 (Glasgow year group)	1,225	4.9
500 (a whole school)	124,750	49.9
2,000	1,999,000	199.9

Neither number shows the spread. Everyone with a few friends and a handful of people with nearly all the ties can have the same mean degree and the same density.

Same density, other people

At .1 a typical node has about 2 ties in a 20-node network and about 200 in a 2,000-node network.

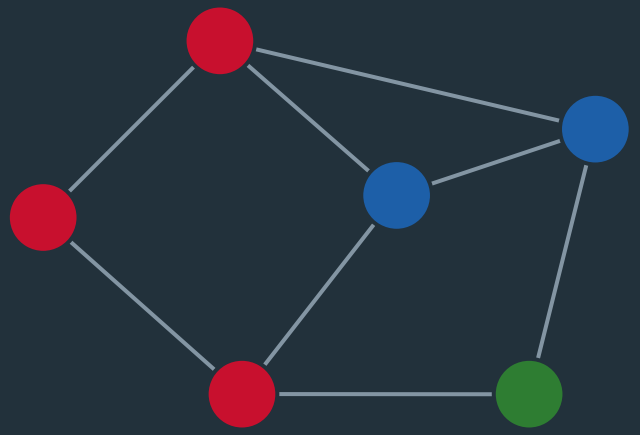
Mean degree stays put

It does not shrink mechanically as n grows. Report it with density.

2

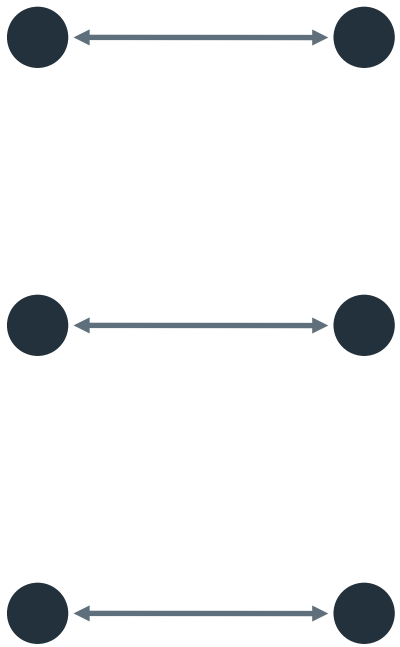
Pairs: the dyad census and reciprocity

20 min · Chapter 6: mutual, asymmetric and null dyads; reciprocity of pairs and of arcs

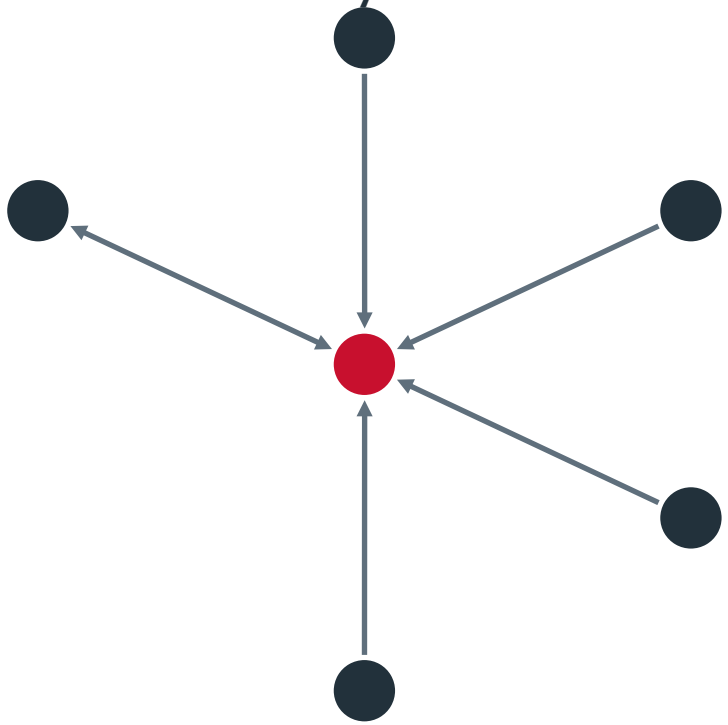


The same density can hide exchange or deference

Three exchanges



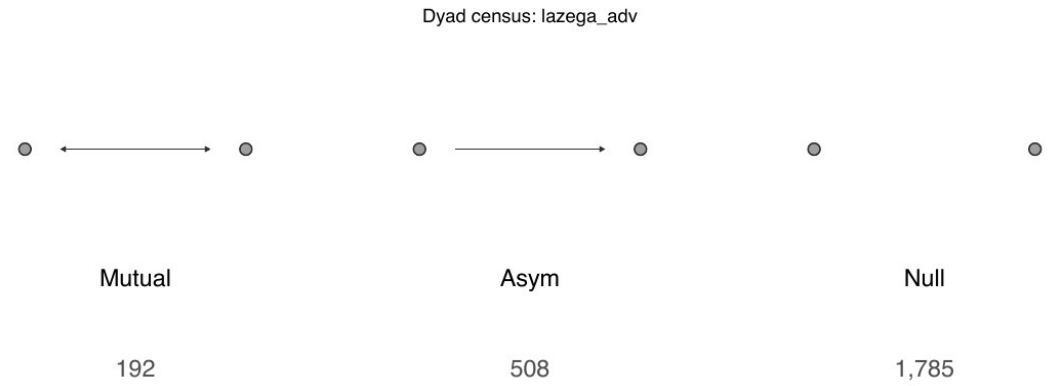
One person consulted by all



Both pictures: 6 arcs among 6 nodes, density 6/30 = .20. Left: 3 mutual dyads, 0 asymmetric. Right: 1 mutual, 4 asymmetric.

The dyad census sorts every pair into one of three states

```
. nwdyads lazega_adv, plot
    Mutual |      Asym |      Null
-----------+-----------+-----------
       192 |       508 |      1785
    Reciprocity: .274
```



.274

of tied pairs are mutual: 192 / (192 + 508)

2,485

pairs in all: 71 × 70 / 2

nwdyads lazega_adv, plot (Chapter 6)

Mutual: both directions. **Asymmetric:** one direction. **Null:** neither. Null dyads stay out of the denominator.

Advice is returned nearly twice as often as friendship: .274 against .148.

Two reciprocities: pairs and arcs

Advice network	Counts	Formula	Value
Pairs (what nwdyads prints)	mutual and asymmetric dyads	192 / (192 + 508)	.274
Arcs (returned nominations)	arcs in mutual dyads	384 / 892	.430

.274

pairs: how many tied pairs exchange

.430

arcs: how many nominations are returned (2M / (2M + A))

Name the denominator. Pairs and nominations are different things; ask which one answers your question.

A mutual dyad holds two arcs, an asymmetric dyad one: 2 × 192 + 508 = 892.

A reciprocity of .27 needs a reference value

It says

Advice is returned in 27 percent of the pairs that have any tie, nearly twice the rate of friendship (15 percent).

It does not say

That 27 percent is high or low, that the exchange is balanced, or that either lawyer thinks of it as an exchange. A shared office or practice can produce mutual ties without any norm.

- **Nomination caps**

  A capped list forces some asymmetry.

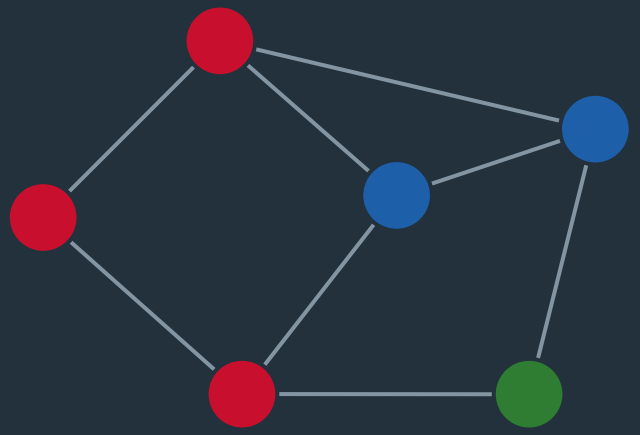
- **Hierarchy**

  A relation hierarchical by construction is asymmetric by construction.

3

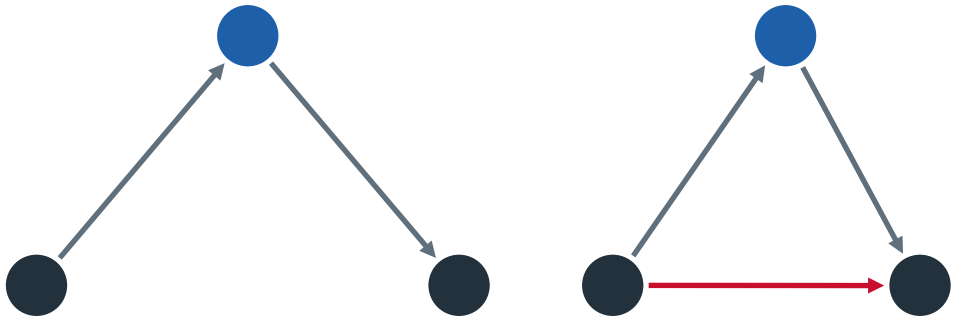
Triples: transitivity and the triad census

25 min · Chapter 6: transitivity, the sixteen triad types, comparing relations against their own density



Transitivity: how many two-paths are closed

```
. nwtriads lazega_adv, plot
    Measure: binary
    Average clustering coefficient: .361
    Global clustering coefficient: .329
    (triad census omitted)
     Transitivity: .329
```



open two-path

closed: i to k present

.329

transitivity: about a third of the directed two-paths are closed

.361

average of the node-wise coefficients

Two other numbers carry the same name. The coefficient of the symmetrized network and the node-wise average give other values; say which one you report.

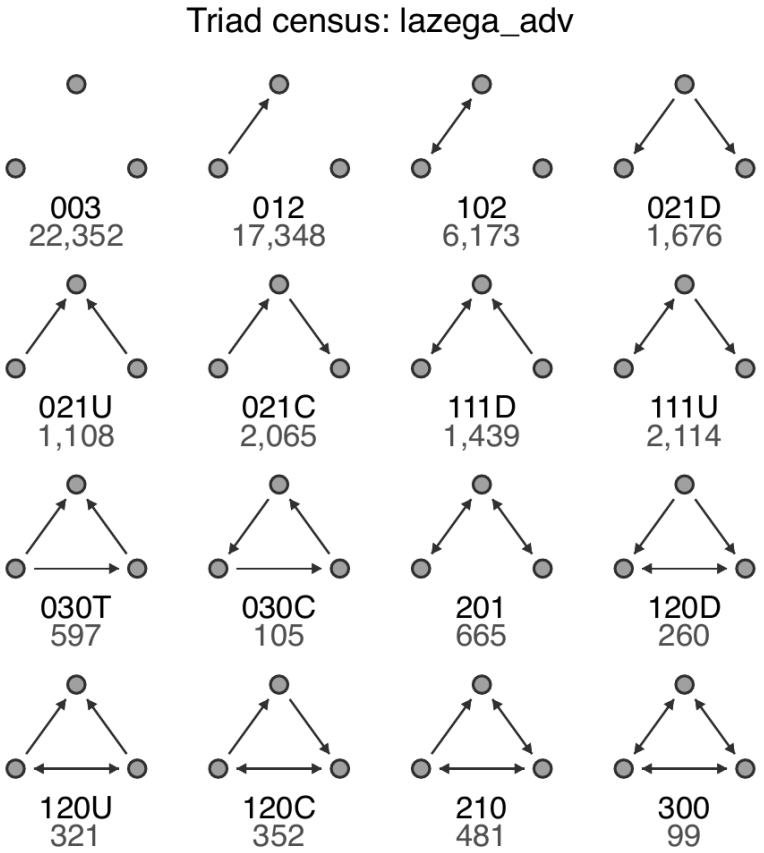
The triad census: sixteen kinds of triple

57,155

triples of 71 lawyers: all sixteen counts add up to it

39%

of the triples (22,352) hold no advice tie at all



Reading a code. 003 has no ties. 102 has one mutual pair and one empty pair. 300 has three mutual pairs. The first digit counts mutual dyads, the second asymmetric, the third null.

The census is where closure meets direction: it separates cycles from transitive triples.

Exchange or ranking? The census says both

Ranking

030T: 597

030C: 105

Almost six transitive triples for each cycle.

Shared advisor

021U: 1,108

Two lawyers both ask a third, who asks neither.

Exchange

102: 6,173

A single mutual pair and an empty pair: the most common triple with a mutual tie.

The descriptive answer is both: a quarter of the connected pairs exchange advice, and the triples show a one-way ordering consistent with a hierarchy.

Compare each relation against its own density

```
. foreach r in adv cow fr {
  2.     quietly nwtriads lazega_`r'
  3.     display "`r': transitivity = " %5.3f r(transitivity)
  4. }
adv: transitivity = 0.329
cow: transitivity = 0.356
fr: transitivity = 0.186
```

Relation	Transitivity	Density	Ratio
Advice	.329	.179	1.8
Co-work	.356	.222	1.6
Friendship	.186	.116	1.6

Friendship is not less clustered, only sparser. Divide by density before comparing relations. The ratio is a reference to the density, not yet a test: Chapter 18 supplies that.

Raw: friendship looks about half as transitive as work. Against density: about the same.

A triangle can come from a shared office as easily as from trust

It says

Advice closes into triangles: transitivity .329, about 1.8 times the density of .179. This is the structure lateral control would need.

It does not say

That the triangles come from trust or watching. Three lawyers who share a corridor or a case form a triangle without any of that. The census does not look at who is in the triangles.

Shared foci

An office or team puts people in contact whatever their friends do.

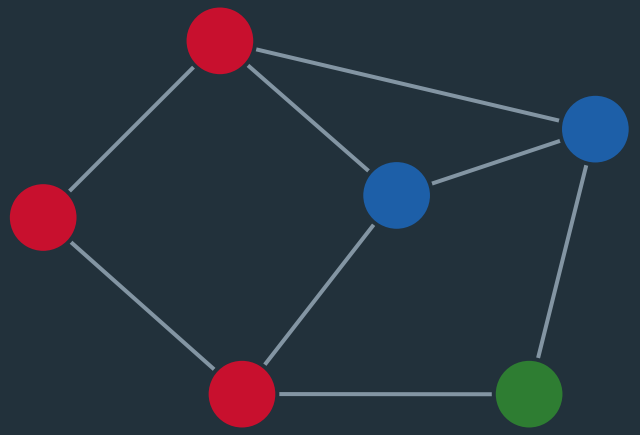
Next question

Do ties stay within offices more than office sizes alone would produce?

4

Mixing: do ties stay within groups?

25 min · Chapter 6: E-I index and its chance value, assortativity



The E-I index: ties across groups against ties within

E-I index

(E - I) / (E + I)

E: ties across categories. I: ties within categories.

Assortativity coefficient

A correlation across ties

Positive when similar actors are tied; for degree or any numeric attribute.

Index near	Reads as
-1	almost all ties stay within a category
+1	ties systematically cross categories
0	not 'no pattern': the chance value depends on the group sizes

Offices are very unequal (48, 19 and 4 lawyers); status is balanced (35 associates, 36 partners).

By status: a weak tendency to stay within the group

```
. nwmixing lazega_adv, attribute(status)
status_ego | status_alter
           | associate   partner      Total
-----------+------------------------------
associate  |       114       162        276
partner    |       216       400        616
Total      |       330       562        892
   E-I Index: -.152   p-value: .02
```

Partners are more active. They send 616 of the 892 advice arcs and receive 562; an associate asks a partner less often (162 arcs, .129) than a partner asks an associate (216 arcs, .171).

Within and across

514 arcs within a status group (114 + 400), 378 across. E-I = (378 - 514) / 892 = -.152.

Friendship

321 of 575 arcs stay within a status group: E-I = -.117.

By office: the pattern is much stronger

```
. nwmixing lazega_adv, attribute(office)
office_ego | office_alter
           |     Boston   Hartford Providence       Total
-----------+----------------------------------------------
Boston     |        539         71         16         626
Hartford   |         86        131          6         223
Providence |         30          9          4          43
   E-I Index: -.511   p-value: .02
```

674 of 892

advice arcs stay within an office (539 + 131 + 4); 218 cross

Hartford and Boston

Hartford lawyers send 131 of 223 advice arcs within Hartford (59 percent), with 19 of the 71 lawyers. Boston: 539 of 626 (86 percent).

Friendship looks the same

E-I index -.503 for friendship, -.511 for advice. A shared office is also a shared corridor and shared cases.

Read each index against its chance value, not against zero

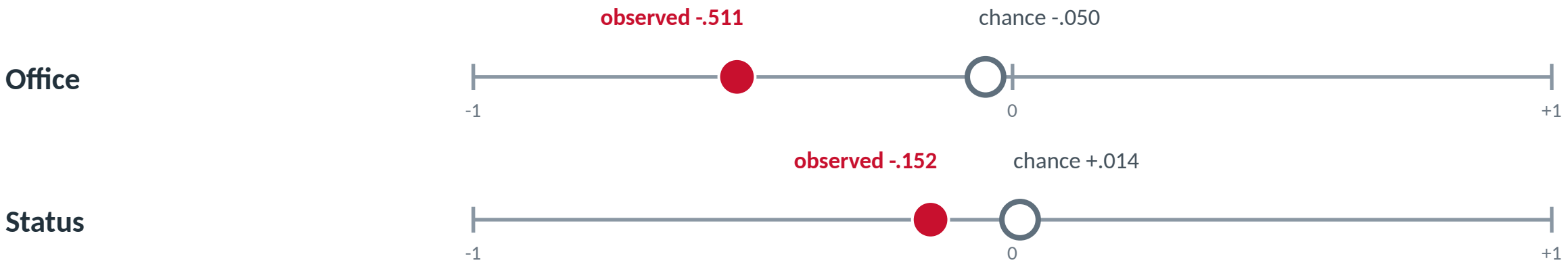
```
. display "status: " (35*34 + 36*35) / (71*70)
status: .49295775
. display "office: " (48*47 + 19*18 + 4*3) / (71*70)
office: .52515091
```

Expected E-I

1 - 2 × chance share

share = sum of n(n-1) over groups, divided by 71 × 70

Observed (red) against chance (grey ring), advice network



The gang: a raw E-I index of .053 looks like no pattern

```
. nwwebuse gang, nwclear
. nwmixing gang, attribute(Birthplace)
   (4 × 4 table omitted: each edge twice)
   E-I Index: .053   p-value: .02
. tab Birthplace
   (four groups, table at right)
```

Birthplace	Members
Caribbean	12
East Africa	6
UK	24
West Africa	12
Total	54

.053

raw E-I index by birthplace: close to zero

Close to zero is not no pattern. The groups are very unequal, so zero is the wrong reference. The next slide computes the right one.

Against its chance value, .053 is strong sorting by birthplace



The chance value

Random labels would put 846 / 2,862 = .296 of the ties inside a group. Expected E-I = 1 - 2 × .296 = +.41.

A floor, not a precise value

With the default 100 permutations the smallest p-value is 2/101 = .02. Report p <= .02.

Against +.41, the observed .053 is far out in the tail: strong evidence that birthplace organizes who co-offends with whom, the opposite of what the raw index suggests.

Assortativity: degree and age among the lawyers

```
. nwassortativity lazega_adv
    Ties: 700
    Assortativity coefficient: -.08
. nwassortativity lazega_adv, attribute(age)
    Assortativity coefficient: .101
. nwassortativity lazega_fr, attribute(age)
    Assortativity coefficient: .118
```

Coefficient	Network, attribute	Value
Degree	Lawyers, advice	-.08
Age	Lawyers, advice	.101
Age	Lawyers, friendship	.118
Degree	Glasgow, wave 1	.172
sport1	Glasgow, wave 1	.069

The output counts 700 ties: pairs with at least one advice arc, not 892 arcs.

Small, but against what? Degree assortativity of -.08 reads as a few much-sought lawyers tied to many less connected ones. Age .10: older lawyers tie to older ones, a little. Whether a value of this size could arise by chance needs a null distribution (Chapters 19 and 20).

Which statistic answers which question

Question	Command	Reports
How many ties relative to the possible number, and how many nodes?	nwsummarize	size, arcs, density, mean degree
Are ties returned?	nwdyads	mutual, asymmetric, null; reciprocity
Do ties close into triangles?	nwtriads	sixteen-type census; transitivity, clustering
Do tied nodes have similar degree or attribute values?	nwassortativity	attribute()
Do ties stay within or cross groups, and by how much?	nwmixing	mixing table, E-I index and permutation p-value

Each number describes one relation in one network. Name both with every number you quote.

Four ways to get it wrong

Density across sizes

The lawyers' .179 and the Glasgow girls' .046 are not a comparison of cohesion. Report mean degree beside density.

E-I against zero

The chance value is about +.41 for the gang's birthplace groups and -.05 for the lawyers' offices. Compare with it, or with the permutation null.

A precise p-value

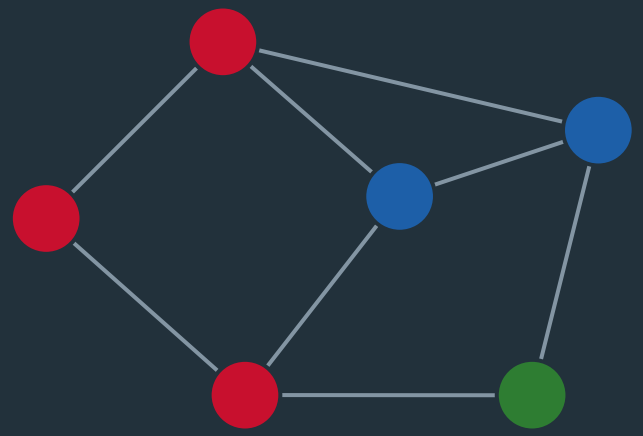
With 100 permutations the smallest p-value is .02 (2/101). Report p <= .02, or ask for more permutations.

Relations mixed in one comparison

Transitivity is .33, .36 or .19 depending on the relation of the firm. Name the network with every number.

Break

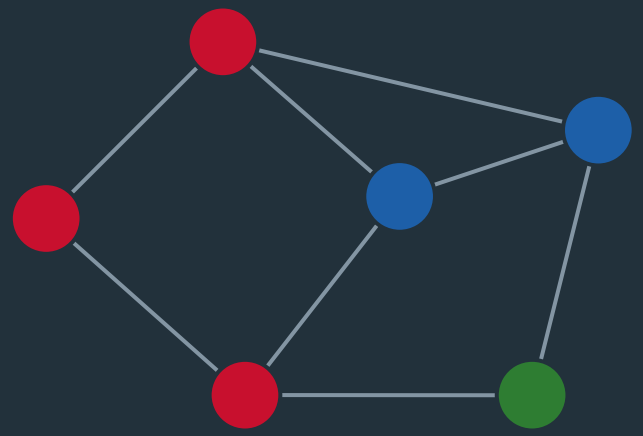
15 minutes



5

Lab: run it, then check your answers

90 min · Chapter 6: commands to run together, four exercises in two rounds, optional exercises



Warm-up: the whole week in six commands

```
. nwwebuse lazega, nwclear
. nwsummarize lazega_adv
. nwdyads lazega_adv
. nwtriads lazega_adv
. nwmixing lazega_adv, attribute(office)
. nwassortativity lazega_adv
. nwassortativity lazega_adv, attribute(age)
```

You should see	Value
Nodes, arcs	71, 892
Mutual dyads	192
Transitivity	.329
E-I index by office	-.511
Degree assortativity	-.08

If a number differs, check that nwclear ran and that the right network name was typed.

Hands-on (round 1, about 25 min)

Ch. 6, Ex. 2

From the friendship census (74 mutual, 427 asymmetric, 1,984 null), recompute by hand the reciprocity of 0.148 that nwdyads prints. Then compute the share of friendship arcs that are returned. Say why the two numbers differ, and which one answers the question "when a lawyer names a colleague as a friend, how often does the colleague name the lawyer back?"

Hint Dyad-based: mutual over mutual plus asymmetric. Arc-based: the arcs that sit in mutual dyads, 2 × 74, over all 575 arcs. The two count different things: pairs and nominations.

Ch. 6, Ex. 5

After nwwebuse lazega, nwclear, run nwdyads and nwtriads on the work network lazega_cow and put it beside advice and friendship in one table with the columns mutual, asymmetric, null, reciprocity and transitivity. Is work more or less reciprocal than advice, and more or less transitive? Then divide each transitivity by the density of its network and say which relation is the most clustered once density is taken into account.

Hint The advice and friendship values are in the chapter; r(transitivity) holds the value after nwtriads, and the densities are 0.179, 0.222 and 0.116. Check that the three dyad counts of each relation add up to 2,485.

Check round 1: pairs, arcs and three relations

Relation	Mutual	Asym.	Null	Reciprocity	Transitivity	Density	Ratio
Advice	192	508	1,785	.274	.329	.179	1.8
Co-work	288	528	1,669	.353	.356	.222	1.6
Friendship	74	427	1,984	.148	.186	.116	1.6

.257

Ex. 2, arcs: 148 / 575 returned friendship nominations

.148

Ex. 2, pairs: 74 / (74 + 427)

Ex. 5. Work is more reciprocal than advice and a little more transitive. Once density is taken into account, advice is the most clustered relation (1.8 against 1.6).

The arc-based .257 answers "how often does the colleague name the lawyer back?"

Hands-on (round 2, about 25 min)

Ch. 6, Ex. 7

The variable gender records 53 men and 18 women among the 71 lawyers. Compute the share of within-gender arcs that random labels would produce, and from it the expected E-I index, as the chapter does for status and office. Then run nwmixing lazega_adv, attribute(gender) and the same for lazega_fr, each after set seed 1 and with the option permutations(1000). Which relation lies further below its expected value, and do the two permutation p-values agree with that reading?

Hint The expected share is (53 · 52 + 18 · 17) / (71 · 70), and the expected E-I index is one minus twice that share. Read each p-value as the chapter reads those for status and office.

Ch. 6, Ex. 11

A researcher compares the density of a 30-person start-up's advice network with that of a 700-person firm. Argue whether density or mean degree should be reported for the comparison, and say in one sentence each what density hides and what mean degree hides.

Hint Work through one example: if everyone in both organizations named 5 colleagues, what would the two densities be? Then ask whether the same mean degree could come from a few people naming everyone.

Check round 2: gender sorts advice more than friendship

Relation	Expected E-I	Observed E-I	p-value (1,000 perm.)
Advice	-.232	-.395	.006
Friendship	-.232	-.329	.062

Ex. 7. Chance share of within-gender arcs: (53 · 52 + 18 · 17) / (71 · 70) = .616, so the expected E-I is -.232. Advice lies further below it (-.395, p = .006); friendship (-.329, p = .062) is not quite significant at 5 percent.

If everyone names 5	Density	Mean degree
30-person start-up	.172	5
700-person firm	.0072	5

Ex. 11. Same mean degree, density 24 times lower in the large firm. Density hides size; mean degree hides the spread.

5 / 29 = .172 and 5 / 699 = .0072: the density falls with n, the mean degree stays 5.

If you finish early: five more exercises

Exercise	Task	You should get
Ch. 6, Ex. 1	From 192, 508 and 1,785 alone, compute n, the arcs and the density.	71, 892, .179
Ch. 6, Ex. 3	Check that the 16 triad counts sum to the number of triples; read the codes 003, 102, 201 and 300.	57,155
Ch. 6, Ex. 4	If each of the six arcs of a triple were present with probability .179, how many triples would be empty or fully mutual?	17,503 and 1.9
Ch. 6, Ex. 6	Rerun nwassortativity with attribute(yrs_frm); larger or smaller than .101 for age?	.089
Ch. 6, Ex. 8	Gang groups 12, 6, 24, 12: expected within-group share and E-I. Why is .053 a poor guide alone?	.296, +.409

Answers to Ex. 1, 3, 4 and 8 are arithmetic; Ex. 4 asks you to compare with the census (22,352 empty, 99 fully mutual).

What to take away from the week

Returned, and ranked

27 percent of the connected pairs are mutual, and one-way transitive triples outnumber cycles almost six to one.

Closed

Transitivity .33, about 1.8 times the .18 that the density alone would give.

Sorted by office

674 of 892 arcs stay within an office; E-I -.51 against about -.05 expected. Status sorts only weakly: -.15 against +.01.

Every number needs a reference value, and zero is not always it. Name the relation and the network with every number.

To read after the session

Before next week

Read Chapter 7, Paths, distance, and reachability

Finish Exercises 2, 5, 7 and 11 of Chapter 6; try 1, 3, 4, 6 and 8

Bring one number from this week and the reference value it needs

Do-files and teaching material: nwcommands.org/book

Instructors are free to use, adapt and share these slides.

